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A new model for nonlinear wind waves. Part 1. Physical model and experimental evidence

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A new interpretation of a nonlinear wind-wave system is proposed. It is proposed that, for steady wind blowing in one direction, a nonlinear wind-wave system can be completely characterized, to a good first approximation, by a single nonlinear wave train having a carrier frequency equal to that of the dominant frequency in the wind-wave spectrum. In this model, the spectral components of the wind-wave system are not considered a random collection of free waves, each obeying the usual dispersion relation, but are effectively non-dispersive bound-wave components of a single dominant wave, travelling at the speed of the dominant wave. To first order, the nonlinear wind-wave system is considered to be a coherent bound-wave system which propagates energy only at the group velocity of the dominant wave and is governed by nonlinear self-interactions of the type found in amplitude-modulated wave trains. The role of short free waves in the system is discussed. Results of laboratory experiments performed by the authors and by Ramamonjiarisoa & Coantic (1976) are found to provide evidence supporting the applicability of such a model to wind waves under virtually all laboratory conditions. Preliminary consideration is given to possible application of the model to oceanic wind waves and conditions are identified for which the model would be most likely to apply.

1. Introduction

During the past two decades, considerable effort has been devoted to the study of nonlinear effects on wind-driven ocean waves. The first nonlinear correction to the linear spectrum was calculated by Tick (1959). Since then, the resonant quartet interaction theory of Phillips (1960, 1961) has been used by Hasselmann (1962, 1963*a*, *b*) and Hasselmann *et al.* (1973) to estimate the nonlinear energy transfer between spectral wave components; this line of investigation has recently been pursued further by Willebrand (1975), West, Thomson & Watson (1974) and Longuet-Higgins (1976).

All these approaches are based upon two assumptions: (i) to a first approximation, the wave spectrum is made up of many linear, random, free wave components and possesses Gaussian or near-Gaussian statistics, and (ii) nonlinear interactions are effective only among wave components which are resonant based on the linear dispersion relation for free waves. In short, the effects of nonlinearity are considered to be weak relative to the effects of randomness.

In this paper, we suggest a model for the characterization of a wind-driven wave field in which the effects of nonlinearity dominate the effects of randomness. We propose that such a nonlinear wind-wave system is well characterized, to a first approximation, by a single coherent nonlinear wave train, with the effect of randomness entering only at the next order of approximation. We have been led to consider this model by examination of results obtained during our recent theoretical and experimental investigations of nonlinear wave pulses and wave trains (Yuen & Lake 1975; Lake, Yuen, Rungaldier & Ferguson 1977), and by results of additional laboratory experiments using wind waves and wave trains (Lake & Yuen 1976; Yuen & Lake 1976), examples of which we report here. We find that, under conditions of fixed fetch and steady wind blowing in one direction, essentially all of the energy in the resulting nonlinear wind-wave system is contained in the bound-wave components of a single dominant wave. To a good approximation, the wind-wave system at a given fetch can be characterized by a single dominant frequency and propagates energy at a single group velocity corresponding to that dominant frequency. The individual components in the wind-wave spectrum do not propagate as free waves and do not obey the usual dispersion relation. We are therefore proposing that the behaviour of nonlinearity-dominated wind waves differs significantly from that of the near-linear waves studied in prior treatments of wind waves, because in the latter case all spectral components are considered to be free waves, i.e. they are assumed to obey the dispersion relation.

The following text provides a brief description of the relevant properties of nonlinear wave trains, the laboratory experimental evidence for our interpretation of the properties of nonlinear wind-driven waves, and a list of properties associated with nonlinearity-dominated wind waves. Part 2 of this study (Yuen & Lake 1979) deals with the application of this physical model to provide a first-order theoretical description of nonlinear wind waves.

2. Properties of nonlinear wave trains

It is well known that a Stokes wave train, i.e. a train of waves with a single fundamental frequency and finite, but uniform, amplitude, has a power spectrum containing a dominant component at the primary or carrier frequency and a series of less energetic components at the frequencies of the harmonics of the carrier. It is also well known that the components are bound-wave components, in that the energy of the system is propagated at a single group velocity. These bound-wave characteristics have usually been considered to be limited to waves which have only harmonics in the spectrum and which propagate without change of form. Waves with a broad continuous spectrum, such as that in figure 1, would not therefore be expected to have bound-wave or non-dispersive characteristics. However, it has been found that a Stokes wave train is unstable to modulational perturbations (Zakharov 1967; Benjamin & Feir 1967) and that the long-time evolution of such a nonlinear wave train leads to strongly modulated wave forms without a loss of coherence (Lake, Yuen, Rungaldier & Ferguson 1977). Furthermore, it has been shown (Lake, Yuen, Rungaldier & Ferguson 1977) that the evolution of a one-dimensional, nonlinear, deep-water wave train is well described quantitatively by the nonlinear Schrödinger equation:

$$i\left(\frac{\partial A}{\partial t}+\frac{\omega_0}{2k_0}\frac{\partial A}{\partial x}\right)-\frac{\omega_0}{8k_0^2}\frac{\partial^2 A}{\partial x^2}-\tfrac{1}{2}\omega_0 k_0^2|A|^2A=0. \tag{1}$$

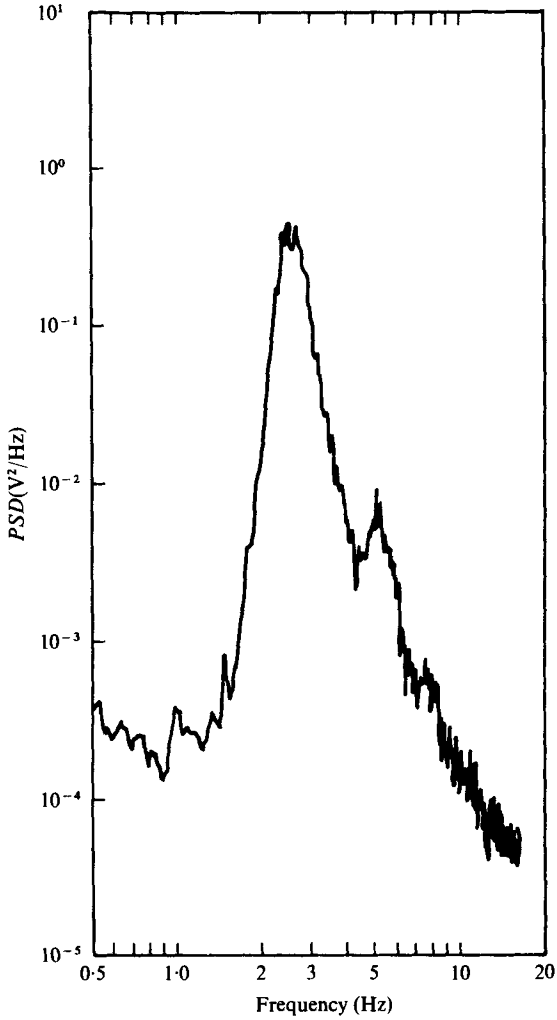

FIGURE 1. Example of power spectrum obtained from amplitude measurements of wind-generated waves. Power spectral density (*PSD*) in $V^2/Hz \propto (\text{amplitude})^2/Hz$ plotted against frequency. Wind speed $u_w = 35$ ft/s; fetch $x = 30$ ft.

This equation, to be satisfied by the complex envelope A of a wave train with a carrier frequency ω_0 (and carrier wavenumber k_0), was first derived by Zakharov (1968). The relationship of A to the free surface $\eta(x,t)$ is given by

$$\eta(x,t) = a(x,t)\cos[(k_0 x - \omega_0 t) + \theta(x,t)], \tag{2}$$

where

$$A(x,t) = a(x,t)\exp[i\theta(x,t)]. \tag{3}$$

In this notation, $A(x,t)$, and hence $a(x,t)$ and $\theta(x,t)$, are all slowly varying functions of x and t; $a(x,t)$ is real and non-negative, representing the wave envelope of the rapidly-varying carrier wave; and the x and t derivatives of θ give the perturbation to the wavenumber and frequency of the carrier wave. According to the set of equations (1)–(3), to leading order, the wave energy is propagated at a constant velocity $C_{g_0} = \omega_0/2k_0$. Similarly, the leading-order propagation speed of the crests is $C_0 = \omega_0/k_0$. These results are hardly surprising for a weakly nonlinear, nearly-uniform wave train.

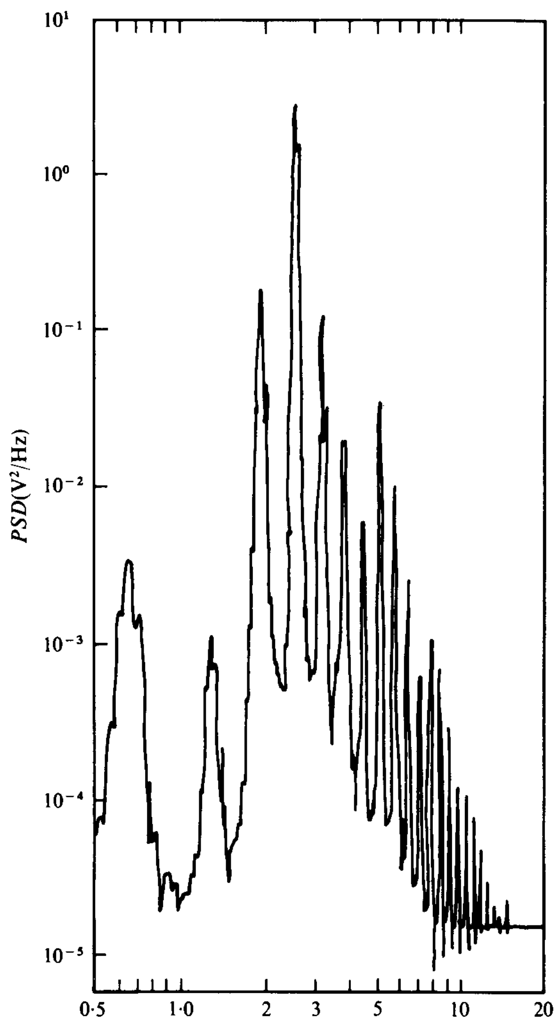

FIGURE 2. Spectrum of amplitude-modulated nonlinear wave train, carrier frequency = 2·5 Hz, initial wave steepness ka = 0·24.

Investigation of the long-time evolution of such nonlinear wave trains (Lake, Yuen, Rungaldier & Ferguson 1977), however, also shows that the nonlinear Schrödinger equation provides a valid description of wave-train properties at stages of evolution where the wave trains are far from uniform. In fact, good quantitative agreement was obtained for cases where the wave trains undergo modulations strong enough to cause local wave breaking. At such strongly-modulated stages of evolution, the power spectra (both experimental and numerical) contain many components in addition to the primary wave and its harmonics. Since the wave train still obeys (1)–(3), these components are not free waves and do not satisfy the free-wave dispersion relation, but are merely Fourier components needed to describe the complicated shapes of the individual waves as well as of their envelope. To leading order the individual components still travel at the single speed $C_0 = \omega_0/k_0$, the phase speed of the carrier wave,

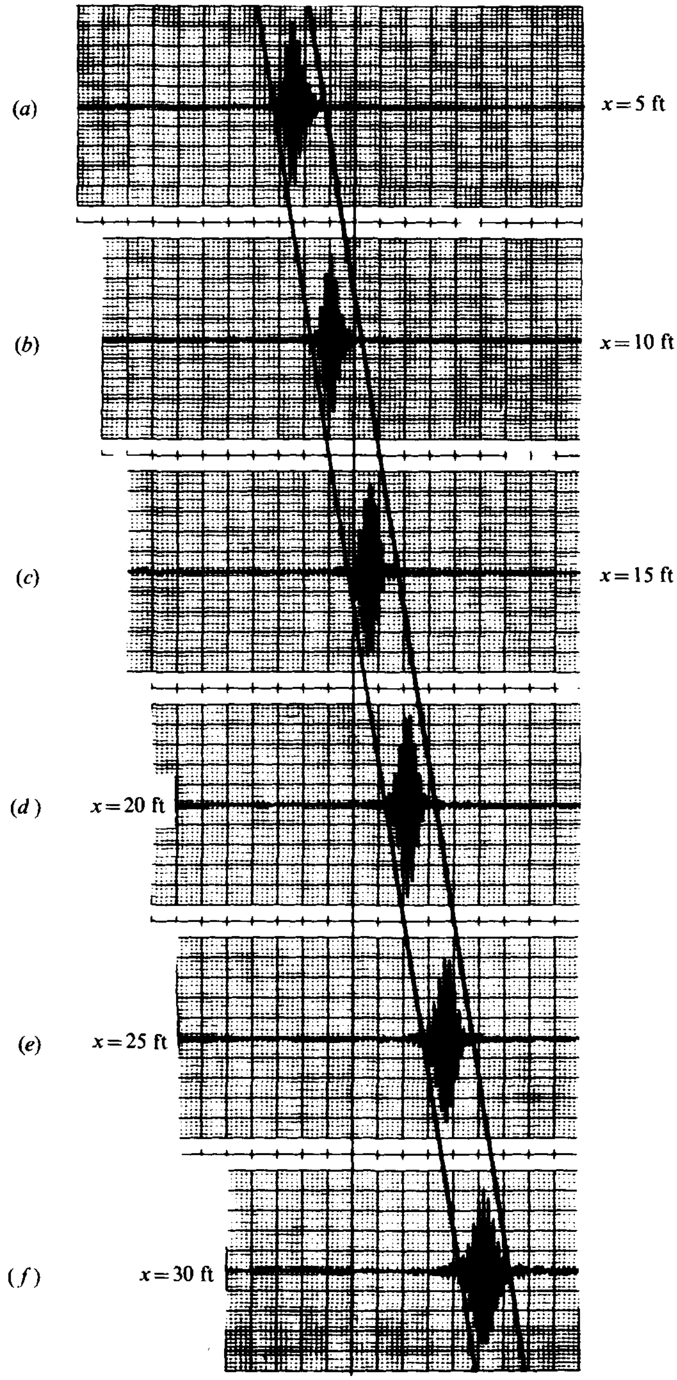

FIGURE 3. Measurements of the propagation of a nonlinear wave pulse (with a spectrum similar to that in figure 2) showing energy propagation at a single speed. (*a*) $x = 5$ ft; (*b*) $x = 10$ ft; (*c*) $x = 15$ ft; (*d*) $x = 20$ ft; (*e*) $x = 25$ ft; (*f*) $x = 30$ ft.

and the energy (or envelope) propagates at the corresponding carrier wave group speed $C_{g_0} = \omega_0/2k_0$. The spectrum shown in figure 2 is an example of the spectrum of such a wave train during a strongly-modulated stage in its evolution. Although the spectrum consists of discrete lines, it covers a relatively broad frequency range, contains many components that are not harmonics of the carrier wave, and in fact the existence of a coherent carrier wave form would not be evident solely from inspection of the spectrum. Nevertheless, the wave system which produced the spectrum in figure 2 is one having a coherent carrier wave, bound-wave components, and a single energy propagation speed, although it does not propagate without change of form. Another example of such a wave system can be seen in measurements of the propagation of nonlinear wave pulses, such as those displayed in figure 3 (Yuen & Lake 1975). The spectra of such pulses contain components over a relatively broad range of frequencies and are very similar to the spectrum shown in figure 2, yet it is clearly evident that the wave system is effectively non-dispersive and that wave energy is propagated at a single speed.

These and other examples from our previous investigations of wave trains and pulses provide evidence that nonlinear self-interactions can produce coherent bound-wave systems which have relatively broad spectra. Inspection of such examples forces one to conclude that propagation of energy at a single speed by a coherent carrier wave in a bound-wave system is not confined to simple wave systems composed only of harmonics and propagating without change of form, and that the existence of a broad continuous wave spectrum of the type shown in figure 1 does not preclude the possibility that the measured wind waves are bound waves with essentially one energy propagation speed.

3. Properties of nonlinear wind waves

3.1. *Laboratory investigations*

A typical power spectrum obtained from wave amplitude measurements in a wind-wave tank is shown in figure 1. Such spectra provide useful representations of the averaged Fourier decomposition of wind-wave measurements, but unless they are used together with phase information, which is required in order to determine the degree of coherence of the wave forms or the nature of the coupling between wave components, they provide a description of the wave system that is incomplete at best and may even be misleading. There is, for example, no information in the spectrum which provides a clear indication as to whether the wave components are bound or free.

Another aspect of wind-wave characteristics, which is very familiar to experimentalists working with wind-wave tanks, is the time history of the wave amplitude at a fixed fetch (e.g. figure 4). This is the raw form of the data that is most commonly obtained in wind-wave measurements. The appearance of a dominant wave scale in such measurements has for many years been relied upon by experimentalists in setting up wind-wave conditions, since a count of the peaks or zero-crossings of the 'dominant wave' in the measured wave form provides a good estimate of the frequency at which the peak of the wind-wave spectrum will appear when the data are later reduced to spectral form. The implications, however, of the appearance of the 'dominant wave' and its observed coherence for interpretations of wind-wave hydro-

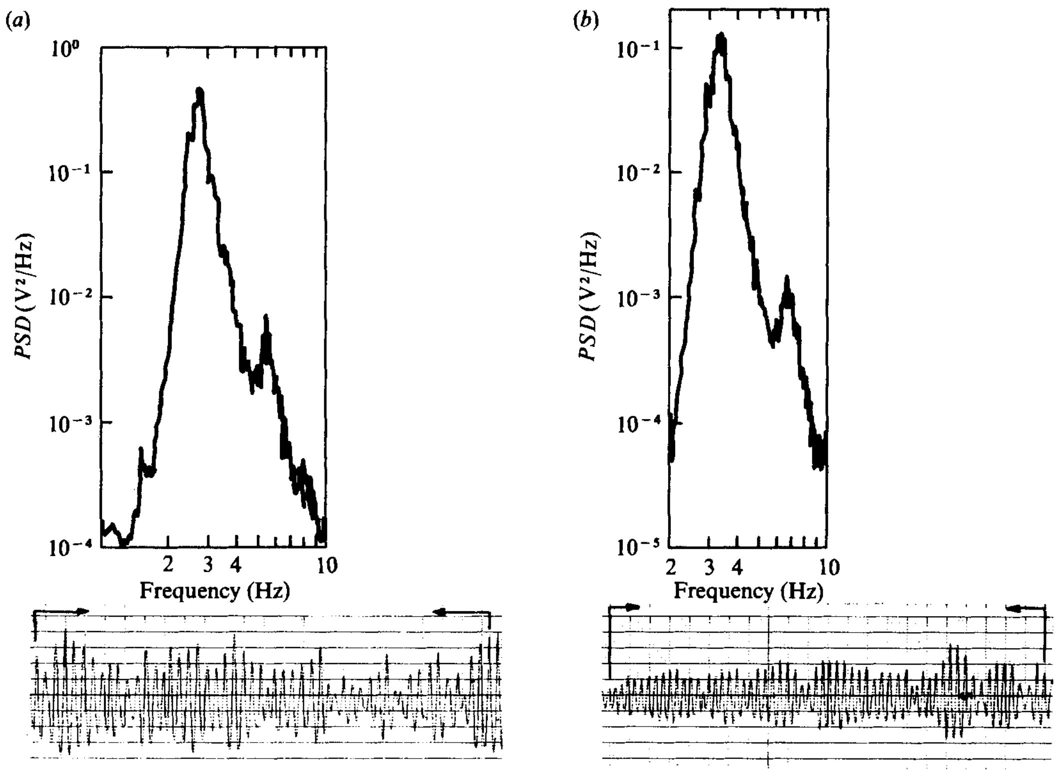

FIGURE 4. Wind-wave amplitude records, each showing evidence of a single carrier frequency (in a wave-counting or zero-crossing sense) at the frequency of the peak in the corresponding spectrum. Arrows on wave forms indicate waves counted. (*a*) $u_w = 30$ ft/s; spectral peak at 2·8 Hz; wave counting:

$$f = \left(\frac{53\,\text{cycles}}{116\,\text{div}}\right)\left(6{\cdot}25\,\frac{\text{div}}{\text{s}}\right) = 2{\cdot}86\,\text{Hz}.$$

(*b*) $u_w = 20$ ft/s; spectral peak at 3·4 Hz; wave counting:

$$f = \left(\frac{61\,\text{cycles}}{110\,\text{div}}\right)\left(6{\cdot}25\,\frac{\text{div}}{\text{s}}\right) = 3{\cdot}47\,\text{Hz}.$$

dynamic characteristics have not been fully exploited. It appeared to us, for example, that the 'dominant wave' might be, in a very real sense, the only true wave in the wind-wave system at that fetch, and that the highly coherent wind-wave system (as seen in the measured wave amplitude records) was actually more like a single amplitude-modulated nonlinear wave train than a superposition of many relatively independent wave components. We therefore tested this proposition experimentally by measuring, analysing and comparing particular characteristics of both nonlinear wave trains and wind-driven waves.

The experiments were performed in a $3 \times 3 \times 43$ ft water tank. A programmable surface wave maker is located at one end of the tank and a wave-absorbing beach at the opposite end. An open-circuit wind tunnel (3×3 ft cross-section) extends over the upper water surface along the 40 ft long working section of the facility. The inlet contraction section of the wind tunnel is located over the wave maker end of the tank, and the diffuser and fan are located downwind of the wave absorber end of the wave tank. The wind tunnel is capable of producing steady wind speed conditions in the facility for a range of wind speeds from 5 to 40 ft/s. The entire wind-wave generation system (inlet, test section, diffuser, fan and motor drive) is mounted directly on the laboratory floor and not on the wave tank itself. The wave tank and wind-wave system are extremely well isolated from one another, the only points of contact between the two being flexible rubber seals which prove the air-seal between the two structures. In addition, the programmable surface wave maker system is mounted on the wind tunnel structure and is therefore also mechanically isolated from the wave tank structure. In the experiments, the evolution of mechanically-generated wave trains and wind-generated waves was measured using capacitance-type, single element, wave amplitude gauges located singly and in pairs aligned in the direction of wave propagation at stations 5, 10, 15, 20, 25 and 30 ft downstream of the wave maker. The gauge outputs were linearly proportional to wave amplitude, with sensitivities typically 3 V/in. over a 2 in. range. Additional details regarding the facilities and procedures used in these experiments may be found in Lewis, Lake & Ko (1974); Yuen & Lake (1975); Lake & Yuen (1976); and Lee (1977).

Frequency of dominant wave. In order to investigate the degree of coherence of the dominant wave in a wind-wave system at fixed fetch, we began an examination of wave amplitude records to identify the extent to which the dominant wave can be considered to be a carrier wave of essentially constant frequency. Since visual inspection of amplitude records from wind-wave measurements indicates that the dominant wave has a highly regular frequency or period, we have used analog and digital processing to determine quantitatively the range of variation of the dominant wave period in wind-wave records obtained under steady wind conditions at a fixed fetch over time periods comparable to those used in obtaining wave spectra (from 10 to 20 min real time). For the analog processing, a Vidar frequency-to-voltage converter is used to obtain a voltage signal which is linearly proportional to the dominant frequency (in a zero-crossing sense) of wind-wave amplitude records which are fed into the device from tape recordings of wave measurements at eight times the real-time recording speed. The device determines the frequency of zero-crossings in the records and converts the result to a voltage that is proportional to the zero-crossing frequency but independent of wave shape (as long as each signal 'crest' or 'trough' exceeds a minimum level that is a few per cent of the peak-to-peak range of the signal). The response time of the device is such that, under the conditions used, the output voltage has an equivalent response time, in dominant-wave periods, of about 10 cycles. This means that the operation tends to average out dominant-wave frequency variations that occur on time scales shorter than about 10 wave periods. The results of such processing (as in figure 5, for example) indicate that for wind-wave records with a dominant-wave frequency near 2·8 Hz, the dominant-wave frequency is constant within about $\pm 0{\cdot}30$ Hz for the length of the record. Note that this range of variation is considerably narrower than the range of components apparent in the spectrum.

In order to obtain a more precisely defined measure of the variation of the frequency or period of the dominant wave, the wave records were digitized and processed by computer. The digitization rate is 250 samples/s for a resolution, in period of the zero-crossings, of 0·004 s. Effects of any long-term d.c. signal drift are eliminated by performing a 4 s sliding mean removal prior to calculation of zero-crossing periods. A

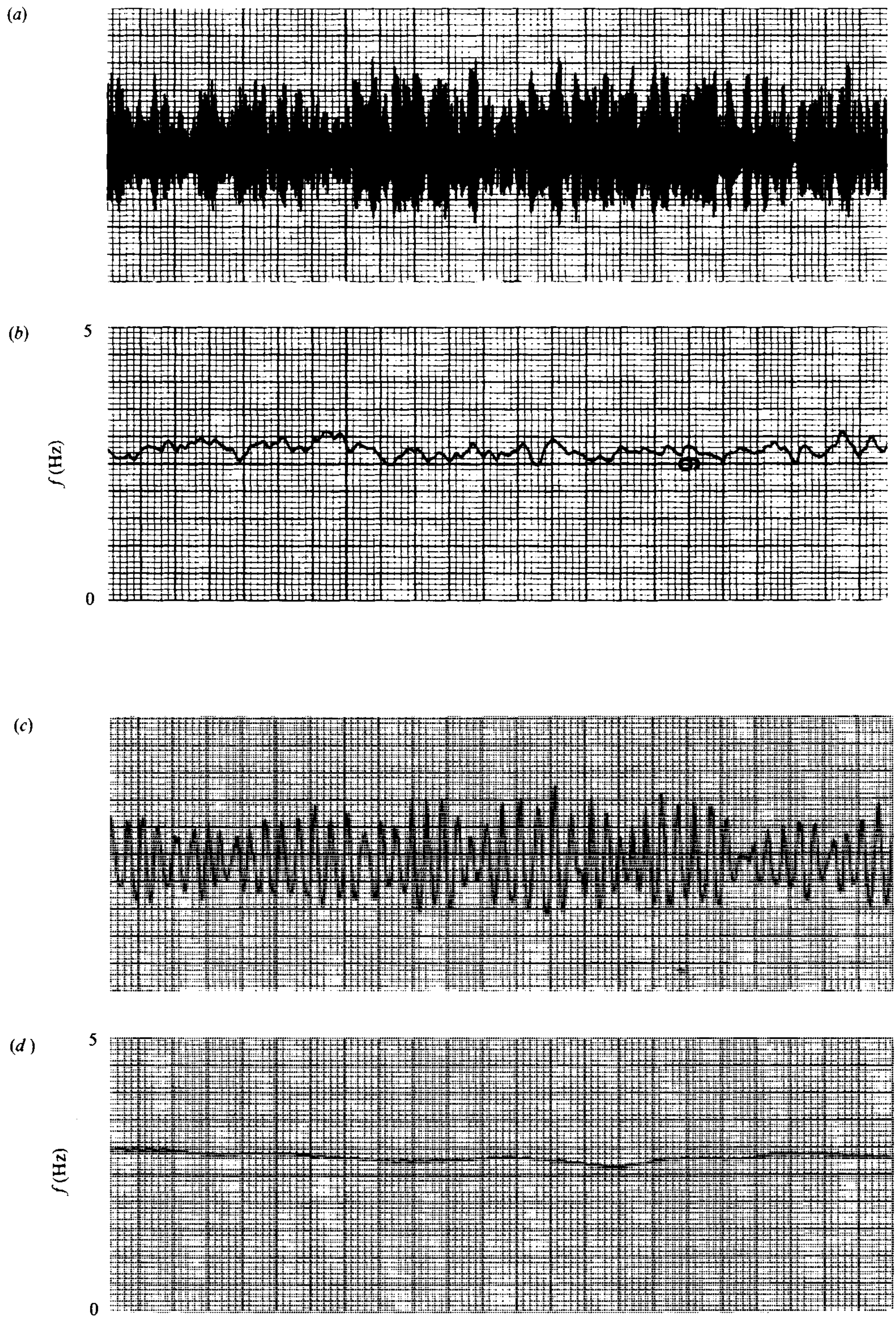

FIGURE 5. Example of analog processing for zero-crossing frequency variations in wind-wave amplitude records. (*a*) Wave amplitude record; (*b*) zero-crossing frequency signal, variation over approximately 475 cycles; (*c*) wave amplitude record; (*d*) zero-crossing frequency signal, variation over approximately 50 cycles.

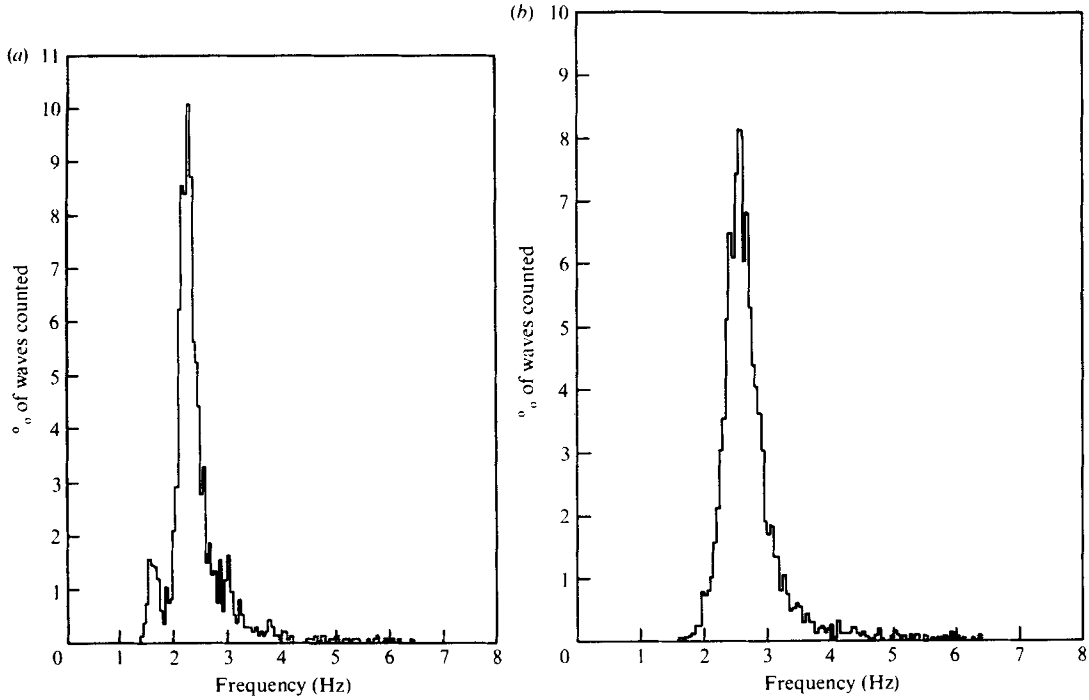

FIGURE 6. (a) Distribution of frequencies obtained from zero-crossing periods of individual waves in the measured wave record of a strongly-modulated nonlinear wave train. $x = 30$ ft; 10 min real-time record. Spectrum and wave form are shown in figure 8. (b) Distribution of frequencies obtained from zero-crossing periods of individual waves in a measured wind-wave record. $u_w = 35$ ft/s; $x = 30$ ft; 10 min real-time record. Spectrum is shown in figure 1 and wave form in figure 9.

zero-crossing is defined as having occurred when a cycle occurs in which the peak and trough exceed a specified minimum level, given as a function of the maximum wave height in the record, so that the periods of waves carrying a specified fraction of the total wave energy can be examined. Thus far, the program has been used to calculate histograms of the periods of zero-crossings in wind-wave records and in modulated wave-train records.

The cases which have been run (as for example in figure 6) indicate that the periods measured for the waves which carry approximately 98 % of the energy in a 2·6 Hz dominant frequency wind-wave spectrum have a range that is greater, by only a fraction of a hertz, than the corresponding range measured for a 2·5 Hz strongly modulated wave train. A greater percentage of the wave-train periods is concentrated at the period corresponding to its dominant frequency than in the wind-wave case, however, so that while the overall range of variation of dominant wind-wave periods is not significantly greater than the range of variation of zero-crossing periods produced by the nonlinearity-induced modulation of a coherent nonlinear wave train, the latter has a slightly more constant carrier frequency. Again, the range of variation of dominant wave frequency obtained from zero-crossing periods is considerably narrower than the range of components apparent in the spectrum (figure 1).

Phase speeds. In addition to finding that coherent bound-wave systems (nonlinear wave trains) can have broad spectra and that wind waves have dominant waves with

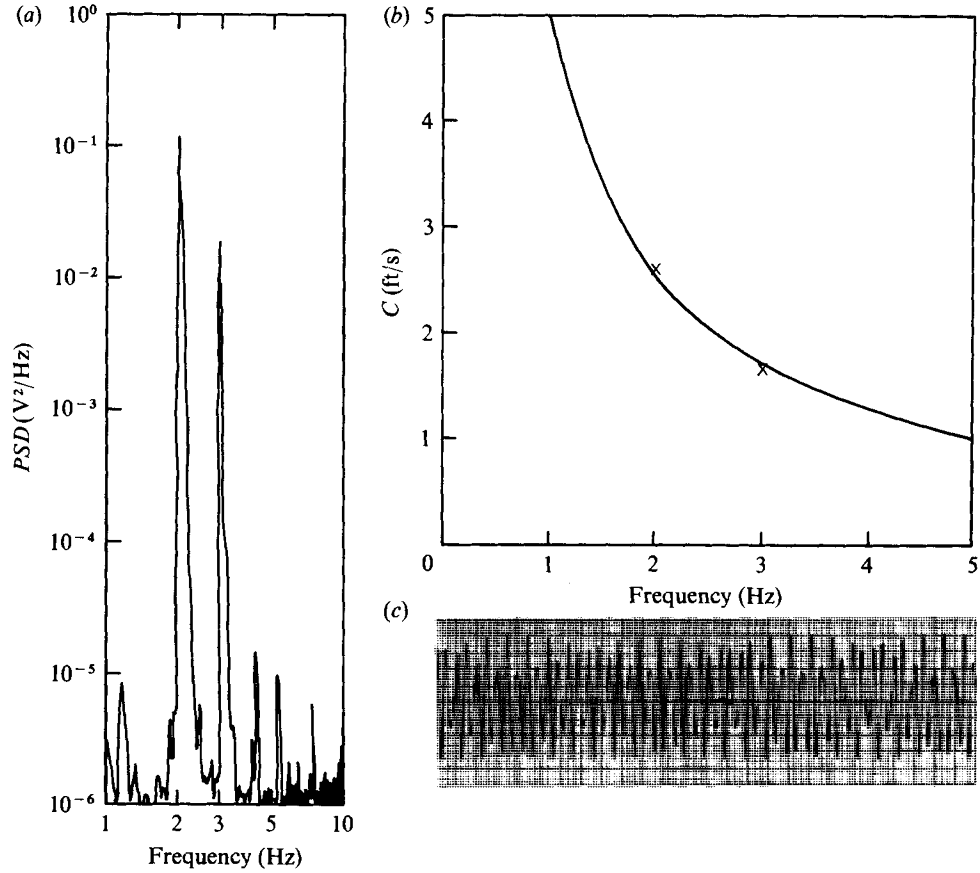

FIGURE 7. Measured phase speeds of frequency components of a linear, multi-frequency wave train. (*a*) Spectrum. (*b*) Phase speed; ——, dispersion relation; ×, experiment. (*c*) Wave amplitude record.

a relatively high degree of coherence, we have also performed experiments to test directly the proposition that the dominant wind wave is the only true wave in the system in the sense that the other significant components in the spectrum are bound-wave components of the dominant wave. This was done by obtaining measurements of the phase speed of individual frequency components in the wave spectrum. Wave amplitudes were measured using two probes separated by a short distance (typically 3 in.) in the direction of wave propagation. The output of each probe was then narrow-bandpass filtered (high pass and low pass set at the same value, both filters having 48 dB/octave cut-offs) and then the two outputs were cross-correlated. The time to maximum cross-correlation and the probe separation were then used to obtain the phase speed of the bandpassed frequency component. This operation was performed for measurements of wave trains with multiple frequency components and infinitesimal amplitudes (linear waves), for nonlinear wave trains (i.e. $ka \gtrsim 0{\cdot}1$) at various stages of modulation, and for wind-driven waves at various wind speeds.

For the case of linear wave trains with multiple components we found that each component travels at its own phase speed, as given by its frequency and the dispersion relation (figure 7). This result demonstrates that the measurement technique does

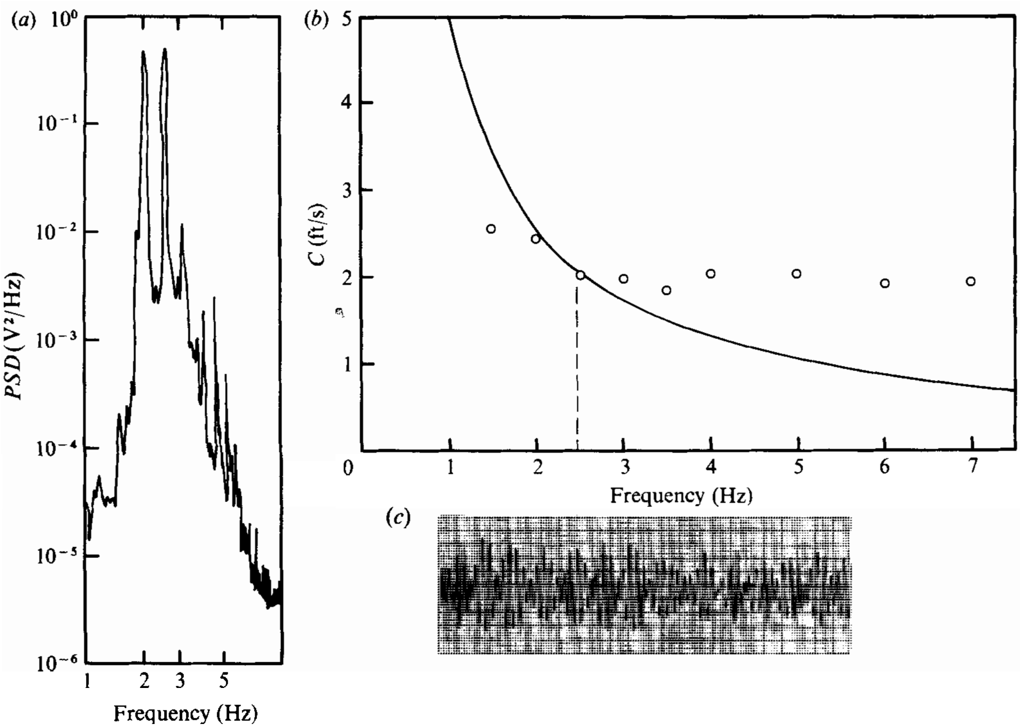

FIGURE 8. Measured phase speeds of frequency components in a strongly-modulated nonlinear wave train. (a) Spectrum. (b) Phase speed; ——, dispersion relation; ○, experiment. (c) Wave amplitude record.

reproduce the expected result that the linear wave trains are composed of free-wave components. For the case of nonlinear wave trains, we found that all components (not simply the harmonics) at frequencies higher than the frequency of the primary or carrier wave travel at approximately a single phase speed – that of the primary frequency component (figure 8). In other words, we found that the speeds of the individual components did not obey the usual dispersion relation and that the components are in fact bound-wave components of the dominant wave (primary component), as expected on the basis of the discussion of nonlinear wave trains in §2. For the case of wind-driven waves, we found that all components having frequencies higher than the frequency of the dominant wave (again, not simply the harmonics) travel at a single phase speed – that of the dominant wave (in this instance equal to the phase speed calculated from the dispersion relation plus a wind-drift contribution, figure 9). In other words, the speeds of the individual components do not obey the usual dispersion relation and the components are in fact bound-wave components of the dominant wind wave.

Our results are not the only measurements of the phase speed of wind-wave components which show this lack of dependence of phase speed on frequency. Wind-wave measurements of this type were first reported by Ramamonjiarisoa & Coantic at the Seattle meeting of the American Meteorological Society in March 1976. They have presented data taken at two wind speeds [also published in French (Ramamonjiarisoa & Coantic 1976)], which show that all components above the dominant frequency

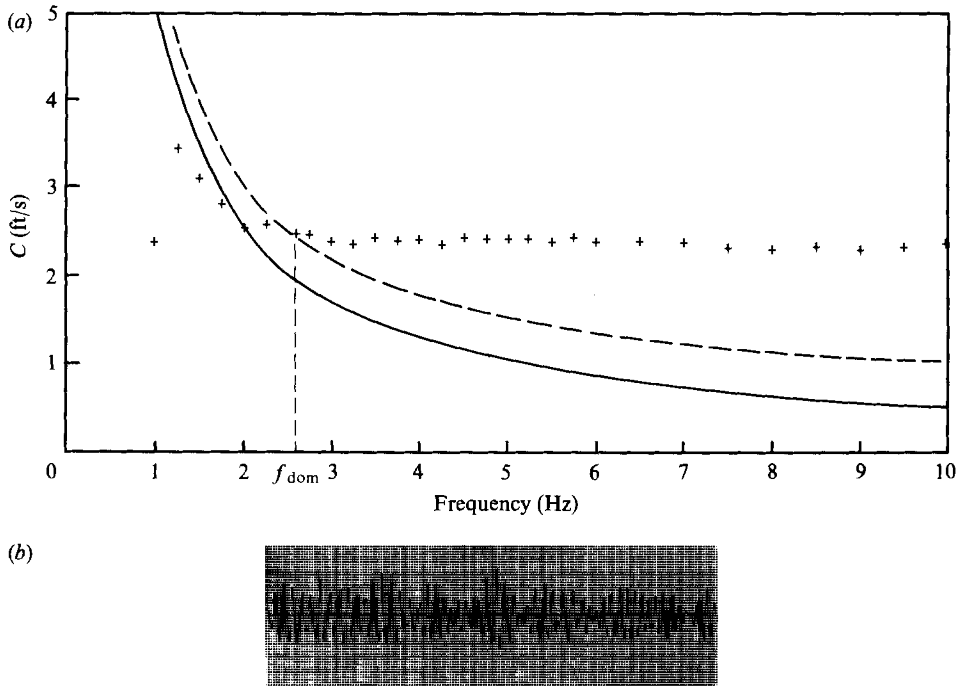

FIGURE 9. Measured phase speeds of frequency components of wind waves, $u_w = 35$ ft/s, $x = 30$ ft. (*a*) Phase speed; ——, dispersion relation; – – –, dispersion relation$+u_d$ ($u_d = C_{dom,\ meas} - C_{dom,\ dispers} = 0{\cdot}51$ ft/s); +, experiment. (*b*) Wave amplitude record. Spectrum is shown in figure 1.

propagate at the phase speed of the dominant wave and they point out that the components seem to be acting as though they are harmonics of the wave even though they are not. We believe that such results, however, when taken together with the rest of our results on nonlinear wave trains and wind waves,† can be used to support the nonlinear wind-wave model we are proposing.

Group velocity measurements. According to our proposed interpretation of nonlinear wind waves, all of the energy in such a wind-driven wave system should propagate at a single speed – the group velocity of the dominant wave. We have also measured group velocities in wind-driven wave systems in order to test directly this aspect of the wind-wave model. Measurements of the group velocity can be obtained by processing the data from the probe pairs that were used to obtain phase speed measurements to the frequency components of wave trains and wind waves. This was done by first squaring (rectification can also be used) the output of each wave amplitude gauge and then low-pass filtering the result to produce a signal that is the square of the envelope of the measured waves. When the envelope signals from the two gauges are cross-correlated, the resultant time to maximum correlation and the separation distance provide a measure of the speed of wave amplitude modulations, or the group velocity. The results for nonlinear wave trains correspond to the group velocity of the primary frequency component, as one would expect. The results for the case of wind

† For example, the results of our wave-train investigations which show that a single nonlinear wave train can have a broad spectrum of components, and that all the spectral components, not simply those that are harmonics of the carrier frequency, can exhibit bound-wave properties.

u_w (ft/s)	$C_{g\,\text{meas}}$ (ft/s)	f_{dom} (Hz)	$\left[\begin{matrix} C_{\text{meas}} \\ \text{at } f_{\text{dom}} \end{matrix}\right]$ (ft/s)	$-\left[\begin{matrix} C_{\text{calc}} \\ \text{from } f_{\text{dom}} \end{matrix}\right]$ (ft/s)	$= u_d$ (ft/s)	$[C_{g\,\text{meas}} - u_d]$ (ft/s)	$\left[\begin{matrix} C_{g\,\text{calc}} \\ \text{from } f_{\text{dom}} \end{matrix}\right]$ (ft/s)
20	1·12	3·2	1·95	1·60	0·35	0·77	0·80
25	1·26	2·9	2·17	1·77	0·40	0·86	0·885
35	1·56	2·6	2·48	1·97	0·51	1·05	0·985

TABLE 1. Examples of comparison of results of measured wind-wave group velocities and group velocities calculated from dominant wave frequency for three wind speeds at fixed fetch.

waves are in agreement with the group velocity of the dominant wave in each case when the wind-drift speed, as determined from the difference between the measured phase speed of the bound-wave system and the phase speed calculated from the dispersion relation using the frequency of the dominant wave, is taken into account (table 1). These results confirm that the energy in the wind-wave system is transported at the group velocity of the dominant wave, as would be expected from our interpretation of wind waves as a nonlinear bound-wave system characterized by a single dominant-wave frequency.

Evolution of dominant waves. The wind-wave characteristics which have been discussed so far are characteristics which apply at a fixed stage, i.e. at fixed fetch or at fixed duration, in the evolution of a one-directional wind-wave system under steady wind. We believe, however, that our proposed use of a modulating nonlinear wave train as a basis for a first-order model of nonlinear wind waves also addresses the frequency/wavenumber evolution aspect of the wind-wave problem. In particular, we have observed and measured a phenomenon of evolving nonlinear wave trains (even without wind effects) which we believe is highly relevant to frequency/wavenumber evolution in nonlinear wind-wave systems. The occurrence of this phenomenon was first reported in Lake, Yuen, Rungaldier & Ferguson (1977). That report is reviewed briefly, and expanded upon somewhat, in the following because we believe the phenomenon we have observed is a truly remarkable result of nonlinear self-interaction in a deep-water wave train, one not predicted by any existing theory for wave-train evolution, and one which is fundamental to frequency/wavenumber evolution in nonlinear wind waves.

In our investigations of nonlinear wave-train evolution, we found that a modulating nonlinear wave train (without wind) will undergo a self-induced shift to a new, lower, carrier frequency whenever further growth of the modulation would require that some waves exceed a maximum realizable steepness. We concluded that the occurrence of the self-induced frequency shift was associated with a steepness limitation for deep-water waves because we observed many cases where wave trains evolved through modulation–demodulation recurrence cycles without a change of carrier frequency, as well as many cases where the wave trains demodulated to new lower carrier frequencies, and the only distinction between the two types of cases which was consistently observed was associated with wave steepness. Frequency changes occurred when the waves in their unmodulated state were already sufficiently steep (e.g. $ka \gtrsim 0{\cdot}3$) that the amplifications required as they passed through the most amplified portion of the wave envelope during the highly modulated state would have required that they attained an unrealizable wave steepness had their wavelength remained fixed. Another admittedly qualitative way of describing what is observed in the wave-form records when this frequency/wavenumber decrease occurs is to say that the modulation becomes so large that a wave crest in each modulation period appears to be reduced to zero amplitude and 'lost' to the wave train as it evolves further and demodulates. Examples of such frequency/wavenumber changes as they appear in wave forms and spectra of amplitude measurements at various stages of wave-train evolution are shown in figures 5 and 6 of Lake, Yuen, Rungaldier & Ferguson (1977) and in figure 10 of this paper. The wave records in figure 10 were made at wave-tank locations which were concentrated primarily in the region where the change in carrier-wave frequency was occurring. The measurements show that as it propagated, the wave train evolved from a weakly modulated condition with a carrier frequency $f_1 = 3{\cdot}25$ Hz and modulation frequency

$$\Delta f = \delta f_1 = (0{\cdot}2)\,(3{\cdot}25\,\mathrm{Hz}) = 0{\cdot}65\,\mathrm{Hz},$$

through a strongly-modulated state and a frequency change, to a nearly demodulated condition with a new carrier frequency of $f_2 = f_1 - \delta f_1 = f_1 - \Delta f = 2{\cdot}60$ Hz. As previously reported, the magnitude of the change, if it occurs, is known from the initial conditions f_1 and ka (since $\delta = ka$ for a Benjamin–Feir modulation instability), and the new carrier frequency is simply the component which was originally the lower of the pair of sideband frequency components which represented the amplitude modulation of the initial wave train. To the extent that average values of ka and the dominant frequency of modulations in wind waves can be identified, the magnitude of such reductions of dominant wave frequency in wind-wave systems should also be predictable.

While the measured wave forms and spectra provide evidence that is highly suggestive of a true changeover of the wave train from a system at one carrier frequency to a system at another lower frequency, a much more demanding test of whether such a changeover has truly occurred is possible because, if it is real, the change of carrier frequency corresponds to measurable changes in phase velocity. In figure 11 the results of measurements of wave-train carrier frequency (obtained from autocorrelations of wave amplitude measurements) and wave-train phase speed (obtained from cross-correlations of wave amplitude measurements using two probes aligned in the direction of wave propagation) are shown. The measurements of each quantity are shown as points, and the lines are the results obtained for each quantity from the measured values of the other quantity using the dispersion relation. The results provide solid evidence that the self-induced change of carrier frequency is real, and indicate that the wave-train frequency and speed satisfy the dispersion relation during, as well as after, the change of carrier frequency.

We believe that this shift to lower sideband frequency is the primary mechanism by which nonlinear wind waves shift to lower frequencies. The shift to lower frequencies is therefore a purely hydrodynamic phenomenon which can also occur in wave trains in the absence of wind if the wave trains become locally 'over-modulated' as they evolve. To first order, therefore, the role of the wind in the evolution of such nonlinear wind waves can be considered as simply a continuing source of energy to the locally dominant wave, which then regularly approaches limiting steepness, inducing the frequency shift mechanism on a continuing basis.

One implication of such a model for the local properties and the evolution of non-

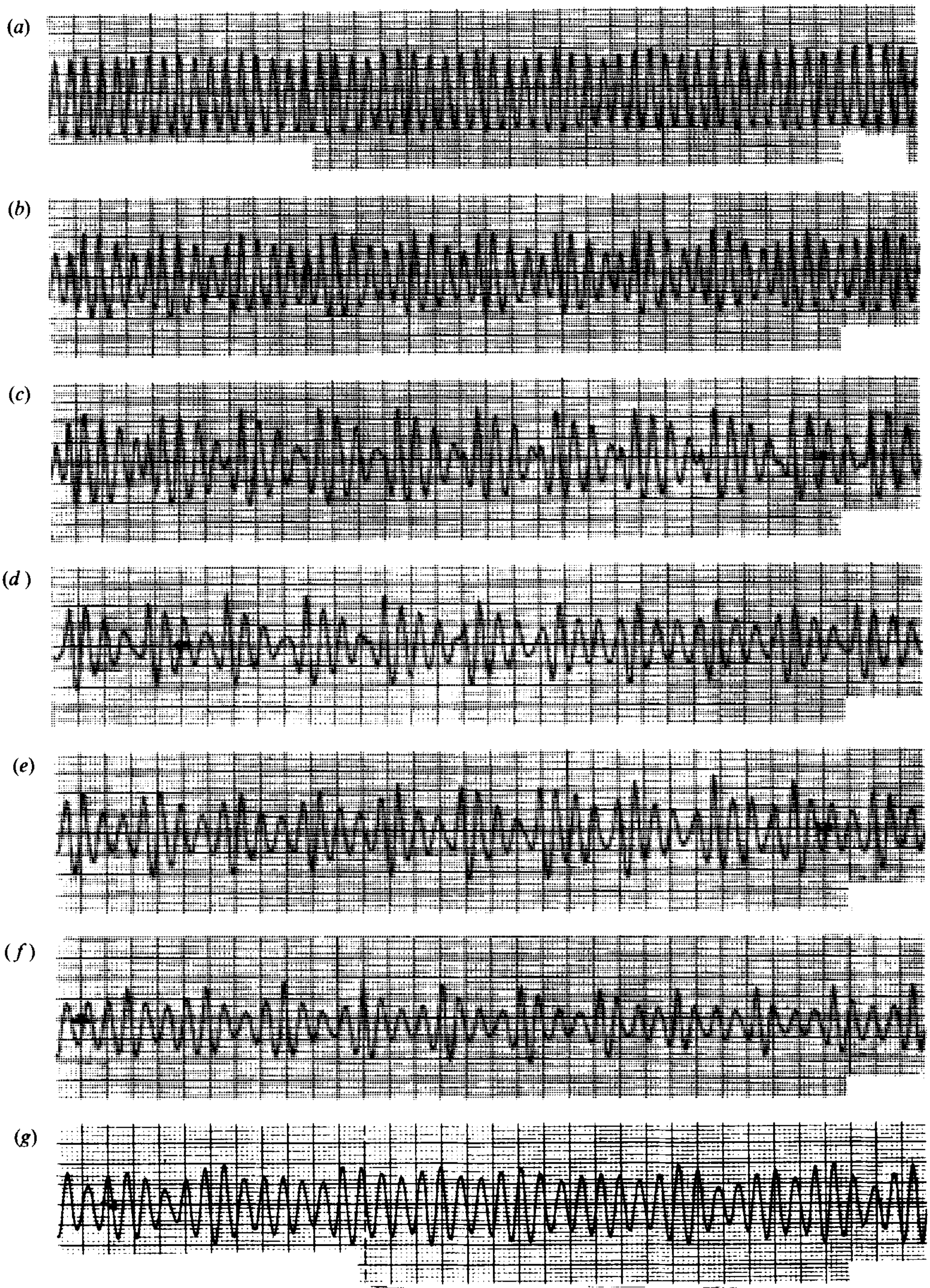

FIGURE 10. Wave-form measurements of a nonlinear wave train during a self-induced change of carrier frequency. (*a*) $f_1 = 3{\cdot}25$ Hz, $\delta = 0{\cdot}2$, $x = 5$ ft; (*b*) $x = 12$ ft; (*c*) $x = 15$ ft; (*d*) $x = 17$ ft; (*e*) $x = 18$ ft; (*f*) $x = 20$ ft; (*g*) $f_2 = f_1 - \delta f_1 = 2{\cdot}60$ Hz; $x = 30$ ft. Note that although the output of every gauge was linearly proportional to wave amplitude, each gauge had a slightly different sensitivity. As a result, the oscillograph record of the output voltage from any given probe is an accurate representation of the wave form of the water waves measured by that probe, but the absolute magnitudes of the wave forms shown in oscillograph records recorded by different probes cannot be used to compare actual wave amplitudes at different measurement stations unless differences in probe sensitivities are taken into account. The series of oscillograph records used in this figure are therefore true representations of the wave-form changes which occur as the wave train evolves, but should not be used to compare wave amplitudes at different measurement stations.

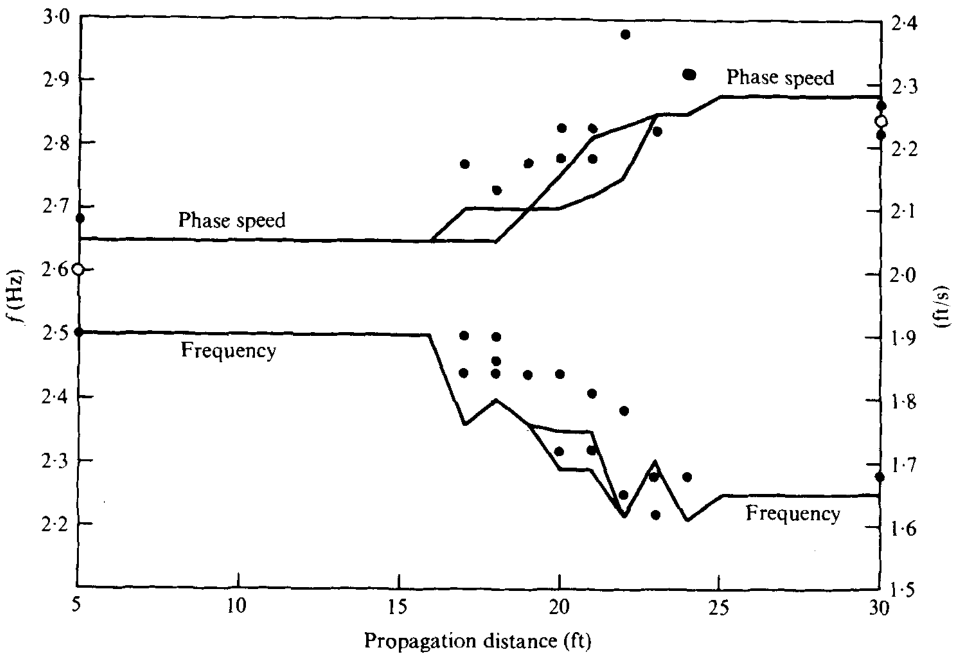

FIGURE 11. Measurement of carrier frequency and phase speed of a nonlinear wave train during a self-induced change of carrier frequency. Note also the two open data points (at the 5 and 30 ft stations), which are phase speeds obtained by doubling values of group velocities obtained from cross-correlations of two-point wave envelope measurements. $f_1 = 2{\cdot}5$ Hz, $\delta = 0{\cdot}1$, $f_2 = f_1 - \delta f_1 = 2{\cdot}25$ Hz.

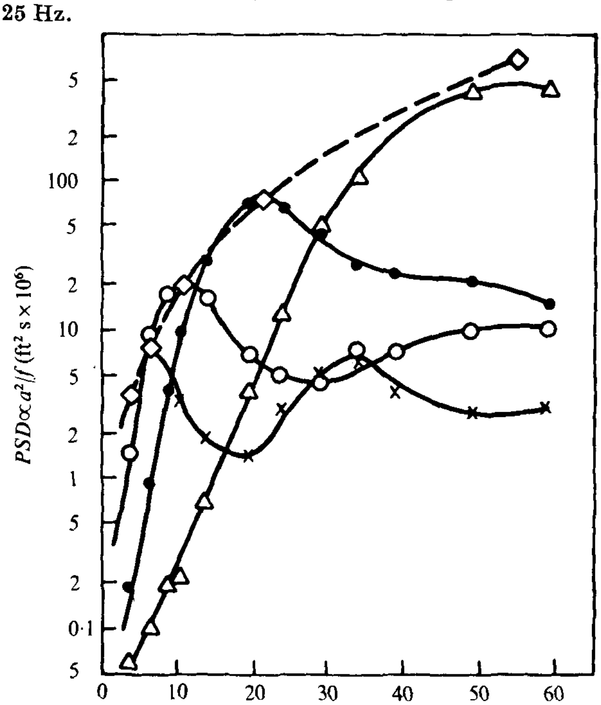

FIGURE 12. Measured growth of wind-wave spectral components with fetch (from Sutherland 1968) and maximum levels obtained using nonlinear wind-wave model with $\bar{k}\bar{a}$ = constant. Solid lines connect experimental data for each frequency. △, 2 Hz; ●, 3 Hz; ○, 4 Hz; ×, 5 Hz. Dashed line is the peak frequency shift prediction obtained from the nonlinear wind-wave model. It was obtained by normalizing to obtain agreement with the data for 5 Hz and it then passes through the maximum energy density level of each frequency component, indicating agreement between the predicted and measured evolution of peak frequency energy density levels.

linear wind waves is that an actively growing wind-wave system under conditions of steady wind regularly approaches a limiting wave steepness, so that on the average the wave steepness, or $\bar{k}\bar{a}$, should be constant as the waves evolve with fetch or duration for fixed wind speed (at least for $u_w/C \gg 1$). Measurements of the average steepness of dominant waves under laboratory (Lake & Rungaldier 1977) and field conditions (as in § 3.2 below) provide evidence in support of such an assumption. Furthermore, the constant mean slope concept and the values obtained from data for $\bar{k}\bar{a}$ are consistent with our assumption that at each fetch the wave field is to first order a single nonlinear modulated wave train. Note, for example, that this means that the amplitude or spectral density of any given frequency component in an evolving wave system will attain its maximum level when that component is the dominant wind wave, and the relationship between the maximum levels of components and their frequencies is given by $\bar{k}\bar{a}$ = constant. This aspect of the model is tested against laboratory measurements of the growth of wind-wave spectral components with fetch by Sutherland (1968) in figure 12, where the relationship between the measured maxima is found to be in good agreement with the relationship predicted by constant wave steepness arguments and the nonlinear wind-wave model.†

Modulations of evolving dominant waves. According to our proposed model, the predominant physical properties of an actively growing wind-wave system can, at any stage of its evolution, be approximated to first order by the properties of a nonlinear wave train having a carrier frequency at the frequency of the dominant wind wave. Another feature of the wind waves which therefore should be worthy of investigation according to the proposed wind-wave model is the character of the amplitude modulations of the wind waves, since the properties of wave trains can be so well characterized in terms of their modulational properties and the evolution of their wave envelopes. The amplitude modulations of wind waves have apparently not often been examined in past investigations of wind-wave properties, but we believe they are a potential source of highly useful information on the basic properties of wind-wave systems.

The modulations can be studied most easily by processing wave amplitude records to produce signals which correspond to the wave forms of the envelope of the wave amplitude, as described previously with regard to group velocity measurements. We have used this processing on wave amplitude records measured over a wide range of laboratory conditions of fetch and wind speed. For each combination of fetch and wind speed, we then analysed the wave envelope records to obtain histograms showing the distribution of zero-crossing frequencies using the digital technique described previously for obtaining zero-crossing frequency distributions of the dominant waves themselves. In this case, however, the zero-crossing frequency distributions provide frequency information on the envelope records, i.e. on the modulation frequencies. In examining such histograms, we find that while there is more of a spread in the envelope

† Since, on dimensional grounds, the maximum value of the spectral density of the wind waves should be proportional to $\bar{a}^2/f$, where f is the frequency of the locally dominant wind wave, and $f^2 \propto \bar{k}$, it follows from the constant wave steepness condition that the maximum value of the spectral density is $\propto f^{-5}$. Although this is the well-known dependence of spectral density upon frequency obtained by Phillips (1958*a*) for an equilibrium spectral range, the result here describes a different phenomenon, namely the relationship between the magnitudes of successive spectral peaks in a series of spectra for evolving wind waves.

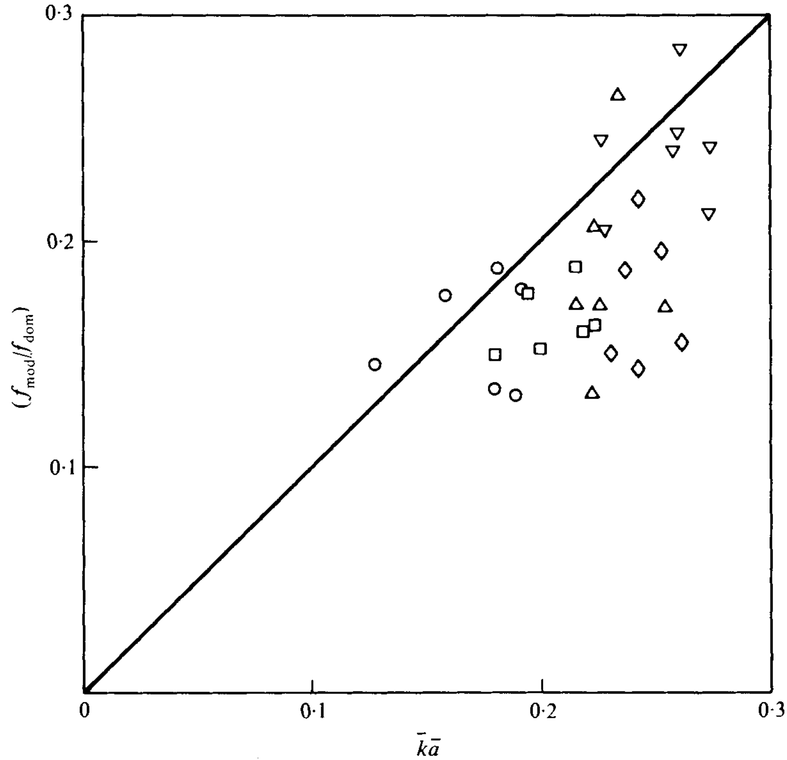

FIGURE 13. Modulation characteristics of wave trains and wind waves. Ratios of modulation frequency and dominant wave frequency plotted against average wave steepness $\bar{k}\bar{a}$. All data points are results obtained by processing amplitude measurements of wind waves for the indicated ranges of wind speed and fetch. The solid line is the wave-train result, the Benjamin–Feir most-unstable modulation, $f_{mod}/f_{dom} = \delta = ka$. 10 ft $\leqslant$ fetch $\leqslant$ 30 ft. u_w: ○, 15 ft/s; □, 20 ft/s; △, 25 ft/s; ◇, 30 ft/s; ▽, 35 ft/s.

frequency distributions than in the dominant-wave frequency distributions, there is also a well-defined, most probable modulation frequency in each case, as would be expected from the nonlinear wind-wave model. Furthermore, there is a direct relationship between dominant modulation frequency and carrier-wave frequency in nonlinear wave trains ($f_{mod}/f_{carrier} = ka$), and so it is of interest to examine the relative values of the dominant modulation frequency, the dominant wave frequency, and the average wave steepness in wind-wave measurements. An example of such a comparison is shown in figure 13 for wind waves measured under laboratory conditions at wind speeds from 15 to 35 ft/s and fetches from 10 to 30 ft. The solid line is the predicted result for a nonlinear wave train and the data points are results from the wind-wave measurements. The data have considerable scatter and there is an indication that the normalized modulation frequency in wind waves is somewhat lower than it would be for a wave train at the same value of $\bar{k}\bar{a}$. Nevertheless, we feel that the fact that a dominant modulation frequency can even be identified for wind waves, and the fact that its relationship to the dominant-wave frequency and the average wave steepness is generally consistent with expectations based on wave-train relationships, is highly supportive of our proposed wind-wave model. We feel also that in future

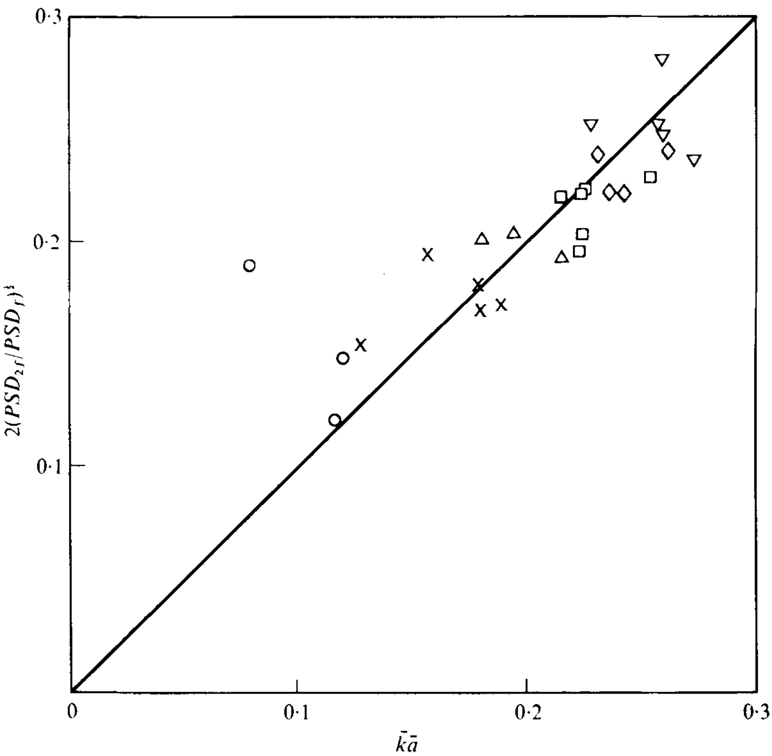

FIGURE 14. Ratios of harmonic-to-carrier amplitude from spectra plotted against average wave steepness $k\bar{a}$. All data points are results obtained by processing amplitude measurements of wind waves for the indicated ranges of wind speed and fetch. The solid line is the Stokes wave result. 10 ft $\leqslant$ fetch $\leqslant$ 30 ft. 10 ft/s $\leqslant u_w \leqslant$ 35 ft/s. u_w: ○, 10 ft/s; ×, 15 ft/s; △, 20 ft/s; □, 25 ft/s; ◇, 30 ft/s; ▽, 35 ft/s.

investigations of nonlinear wind waves, whether in the laboratory or in the field, much more emphasis should be placed on identification of the modulational properties of the waves and on their evolution in terms of wave envelopes.

Unsteadiness of evolving dominant waves. In making comparisons between wind-wave properties and those of modulating nonlinear wave trains, we have also examined spectra in order to determine whether the relative spectral densities of particular frequency components are related in a way that is consistent with a wave-train model. We have calculated the ratio of the spectral density at twice the dominant frequency and the spectral density at the dominant frequency from spectra of laboratory wind-wave measurements for a wide range of wind speeds and fetches. This ratio is of interest because in the most idealized case of a nonlinear deep-water wave, a Stokes wave, it is determined for constant bandwidth simply by the wave steepness as

$$PSD_{2f}/PSD_f = (a_2/a_1)^2 = \tfrac{1}{4}(ka)^2,$$

and because in the relatively high resolution spectra which are possible under laboratory conditions it is usually possible to identify a secondary peak in the spectrum at twice the dominant frequency. In the laboratory, where we already have good evidence that the wind waves are nonlinear in the sense of our proposed model, this examination of spectral levels against wave steepness is a test of whether a useful relationship exists for nonlinear wind waves. As can be seen in figure 14, where the results are plotted, the laboratory wind-wave ratios fall very close to the straight line which identifies the simple Stokes relationship. Since the results indicate that the relationship between spectral levels of harmonics in nonlinear wind waves may be close to that for simple nonlinear Stokes waves, they provide more evidence for the wave-train model and they indicate further that where other data are limited but wave amplitude spectra are available, a crude consistency test for the importance of nonlinear effects might be possible by use of this spectral ratio (whether or not secondary peaks can be resolved) to estimate average wave steepness $\overline{k}\overline{a}$. The indicated steepness should be greater than about 0·1 if significant nonlinear effects are to be expected.

Despite the good agreement between the measured and the Stokes wave values of harmonic spectral levels in figure 14, however, it is important to note here that the comparison cannot be interpreted as indicating that wind waves are like Stokes waves in anything but the grossest average sense. First, as has been shown by Zakharov (1967) and by Benjamin & Feir (1967), a train of Stokes waves of even uniform amplitude is unstable to any infinitesimal perturbations within a range of frequency components centred around the carrier frequency. As a result of this instability, nonlinear wave trains on deep water become amplitude modulated, going through cycles of modulation and demodulation as they evolve (Lake, Yuen, Rungaldier & Ferguson 1977). Perhaps the most pronounced characteristic of nonlinear deep-water waves, whether they are envelope pulses, continuous wave trains or wind waves, is that they are highly amplitude modulated and as a result are highly unsteady waves. This becomes obvious when one realizes that the individual waves in such systems propagate through the modulation envelopes at one-half their phase speed. Since modulation lengths are typically only a few carrier waves long (number of waves/modulation $\propto 2/ka$), and the modulations often become very large, individual waves may typically change amplitude and steepness by a factor of two or more during the time it takes them to propagate only a few of their own wavelengths. Such waves are highly unsteady, their profiles are asymmetric [note the levels of successive troughs in figure 5 of Lake, Yuen, Rungaldier & Ferguson (1977) for example], their particle velocities are drastically different from those of steady waves at the same steepness (Yuen 1977), their breaking characteristics are greatly altered (Yuen 1977; Lake & Rungaldier 1978), and their spectra contain many components which are not simply harmonics of the carrier (figures 1, 2, 4, and 8). Even the modulation envelopes themselves are unsteady, evolving to an end-state that is (in the absence of dissipative effects) neither random nor steady but is instead a continuing series of Fermi–Pasta–Ulam recurrence cycles, as shown by Lake, Yuen, Rungaldier & Ferguson (1977). The differences between the individual waves in nonlinear deep-water wave trains and steady Stokes waves are therefore fundamental. For this reason we do not feel it is appropriate to use a steady-state Stokes wave description of individual wave properties as a basis for modelling the characteristics and evolution of highly modulated and highly unsteady wave systems such as nonlinear wave trains and wind waves on deep water. The most useful model for the evolution and characteristics of such a system appears to be one which describes the system in terms of its envelope dynamics and therefore one based on the nonlinear Schrödinger equation. This appears particularly promising since Lake, Yuen, Rungaldier & Ferguson (1977) have found that the

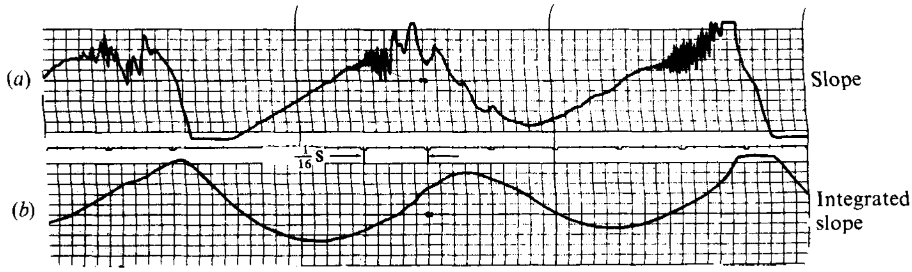

FIGURE 15. Example of slope gauge measurement of wind waves, $u_w = 20\,\text{ft/s}$, $x = 30\,\text{ft}$. (*a*) Slope; (*b*) integrated slope. [From Chang & Wagner (1976), linear vertical scales, the front faces of the waves are to the left of the wave crests in the integrated slope traces.]

evolution of the envelope is well described by that equation with an added dissipative term even when waves become so steep that they break, and investigations by Alber & Saffman (1977) and by Crawford, Saffman & Yuen (1977) indicate that the effects of randomness in such systems can be taken into account using the nonlinear Schrödinger equation as a starting point.

The role of free waves. Although our model for nonlinear wind waves assumes that all of the energy in the wind-wave system is carried by the bound-wave components of a single dominant wave, we are not proposing that there are no free waves (i.e. real waves that satisfy the dispersion relation and are not bound-wave components of the dominant wave) in such a wind-wave system. We do propose, however, that the free waves in the system are primarily short waves which exist on the surface of the dominant wave and contain a negligible portion of the total wind-wave energy. Free waves of this type, although they would tend to dissipate rapidly, can be expected to exist along the surface of the dominant wave because of continuous local-generation by wind action, as well as by capillary generation and breaking associated with steep gravity waves. Because they contain a negligible portion of the energy in the wind-wave system and are short wavelength (high frequency) waves, they are difficult to resolve in most wave amplitude records. In fact, it appears quite possible, in view of the limitations of typical wave amplitude gauges for measurement of waves with very small amplitudes and very short length scales [for example, waves with wavelengths $\leqslant 5\,\text{cm}$ using laboratory-type capacitance gauges (Sturm & Sorrell 1973)], that the high frequency content of wind-wave measurements made using such gauges is almost entirely associated with the high frequency components of the dominant-wave shape. In other words, the spectra obtained from most measurements of wind-wave amplitudes actually may be representative of energy in the bound-wave components of the dominant wave, even at high frequencies.

The physical existence of the high frequency free waves can be detected, however, by high resolution slope gauges or microwave radars, as shown in the example of figure 15. The measurements shown in figure 15 were made by Chang & Wagner (1976) using a high resolution (0·2 mm spatial resolution) laser slope gauge in a wind-wave tank; (*a*) shows the measured slope at a point as a function of time, and (*b*) shows an integrated slope signal, which has been found to be in good agreement with direct measurements of wave amplitude for such wave conditions. The integrated slope output is typical of amplitude measurements of wind waves in that it shows effectively only the dominant wave which contains essentially all of the wave energy. Measurements of the short free waves which exist on the dominant wind wave have also been made by Lee (1977), who used microwave radar Doppler spectra to measure the short-wave speeds. He found the speeds to be equal to the appropriate free-wave phase speed plus contributions from the wind-drift current and the dominant-wave orbital velocity.

Since the properties of the short free waves can be strongly affected by the long dominant wind wave on which they exist, they can be important sources of information regarding the characteristics of the dominant wind-wave system. The distribution of short waves along the dominant wave is not uniform or random. Under conditions of steady wind blowing in one direction, the distribution of the short waves is correlated to a large extent with the phase of the dominant wave, even though they are not bound-wave components of it. Some evidence that this is true, at least under laboratory conditions, is already available. The measurements of both Chang & Wagner (1976, as shown in figure 15) and Lee (1977) show that the short free waves are located primarily in the vicinity of the crest and on the front face of the dominant wave. The dominant wave also affects the speeds and frequencies of the short free waves. The microwave measurements of Lee (1977) have already demonstrated this effect by showing that when short waves exist on a long wave, the net short-wave speed includes a contribution from the orbital velocity of the long wave, and that this contribution can be surprisingly large because there is a preferred location for the short waves on the long wave.

3.2. *Nonlinear wind waves in the ocean*

This description of wind-wave characteristics requires that the wave system have a recognizably coherent dominant or carrier wave propagating predominantly in one direction, and that the dominant wave have sufficient steepness for the wave system to be governed by nonlinear self-interactions such as occur in nonlinear wave trains. The measurements of zero-crossing frequencies, phase speeds of spectral components, and group velocities indicate that wind waves in laboratory facilities (for 10 ft/s $\leqslant u_w \leqslant$ 30 ft/s in our experiments) meet the requirements for application of our model for nonlinear wind waves.

The obvious next question to ask is whether this nonlinear model is appropriate to any of the waves found on the ocean and, if so, whether it applies only occasionally (in time and space) or whether it is the typical or even the predominant situation. While we do not believe that this question can be reliably answered in any quantitative fashion at this time, and our own work is limited to theoretical and laboratory experimental investigations, we have given the question some preliminary consideration.

If we restrict consideration to special cases where conditions are such that the wind and the waves are steady and moving in one direction, the following observations can be made. In the case of actively developing wind-wave systems (in either the fetch-limited or duration-limited sense), our description should apply to at least those wind-wave conditions where the dominant-wave speed is considerably lower than the wind speed, inasmuch as these are the conditions that exist in laboratory facilities. For consideration of other wind-wave conditions, it is necessary to examine the expected hydrodynamic characteristics of the wind waves, in particular, whether

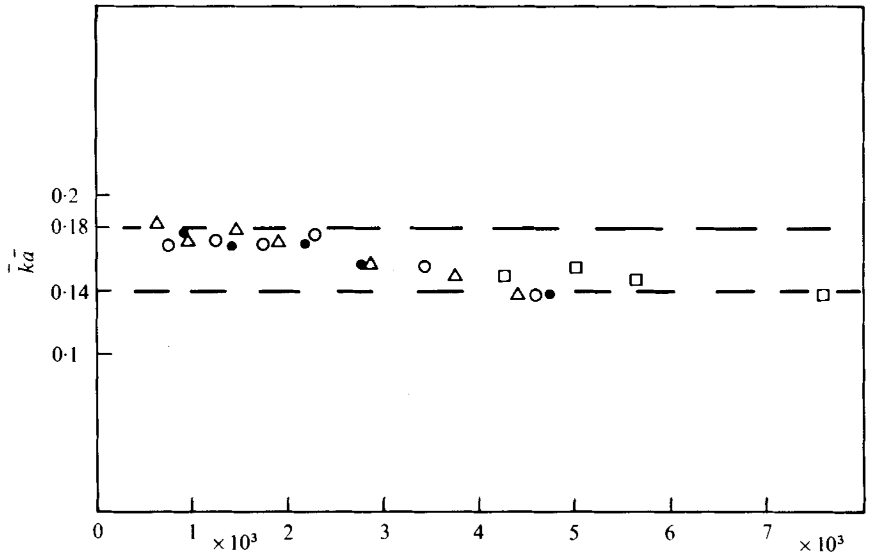

FIGURE 16. Values of dominant wave steepness $\overline{k}\overline{a}$, obtained from field measurements of average periods and amplitudes of dominant waves. Data from Groen & Dorrestein (1958) and Pierson, Neumann & James (1955). u_w = 10, 15, 20 m/s; 0·5 ⩽ duration ⩽ 12 h.

there is a one-directional dominant wave which exhibits coherence and whether the average steepness, or $\overline{k}\overline{a}$, of the dominant wave is sufficiently large. Examination of data from oceanic wind-wave measurements reveals that dominant waves with notable coherence have been observed, and that in some cases tabulations of average periods and amplitudes of the dominant waves are available. In the laboratory we find that a wave train behaves as a nonlinear bound-wave system when the average *ka* exceeds about 0·1 (Lake, Yuen, Rungaldier & Ferguson 1977), whether or not there is wind present. On this basis we find, from a preliminary examination of ocean wave data, that relatively well-developed wind-wave systems which are still growing, but which correspond to conditions where the dominant-wave speed and the wind speed are becoming comparable (wind-wave conditions not realizable in laboratory facilities), also appear to be consistent with our nonlinear model. For example, data from Pierson, Neumann & James (1955) for dominant-wave amplitudes and periods in a 20 knot wind at durations of 4, 6, 8 and 12 h show $\overline{k}\overline{a} = 0{\cdot}14\text{–}0{\cdot}155$ in all cases. Also, an extensive survey of data† on oceanic wind waves by Groen & Dorrestein (1958), as quoted by Stewart (1961), shows that for a wide range of conditions (10 m/s ⩽ u_w ⩽ 20 m/s and durations of from 0·5 h to 12·0 h) the values of $\overline{k}\overline{a}$ were between 0·14 and 0·18. These data are shown in figure 16. If one makes the admittedly strong assumption that these dominant waves propagate in one direction, in addition to being under steady wind and steep as indicated by the measurements of wave amplitudes and periods used in figure 16, they would appear to be of a type describable by nonlinear wave properties.

† Stewart (1961) states that the compilation by Groen & Dorrestein (1958) 'used virtually all the data on wind-waves published prior to 1957 to obtain average wave characteristics as a function of wind duration and speed'.

It should be noted that the conditions which are sufficient to indicate that the wave systems are as described by our model are strictly hydrodynamic. It appears to us that as long as these hydrodynamic conditions of existence of a one-directional coherent dominant wave with $\bar{k}\bar{a} \gtrsim 0{\cdot}1$ are met, the wave system will behave as a nonlinear bound-wave system regardless of wind speed or wave age. (Eventually, however, the reduction or absence of wind energy input and the continued action of dissipation would lead to a wind-wave system that would no longer be sufficiently nonlinear to behave as a bound-wave system.) On the other hand, we recognize that conditions characterized by existence of a steep one-directional coherent dominant wave are not necessarily typical of wave conditions on the ocean surface and that under many circumstances the ocean state may be too linear (i.e. waves too flat, $\bar{k}\bar{a} < 0{\cdot}1$), or too multi-dimensional, or too multi-sourced for this nonlinear model to apply. We do feel, however, that under certain conditions the properties of ocean waves may be characterized more by the nonlinearity of the dominant wave than by a superposition of essentially independent linear wave components. Even in cases where waves are too multi-dimensional or multi-sourced for our one-dimensional nonlinear model to apply, we believe that the possible application of two or more interacting nonlinear dominant-wave systems may be appropriate when constructing models for the dynamics of such systems and that consideration need not therefore be limited solely to models based on superposition of many dispersive free-wave components.

We also recognize that this brief discussion of the possible application of this nonlinear model to the description of properties of wind waves on the ocean is preliminary and incomplete. Continued examination of oceanic wave data, as well as perhaps the performance of some new field measurements to test for bound-wave characteristics, is necessary in order to assess fully the applicability of this nonlinear wind-wave model to oceanic wind waves.

4. Conclusions and discussion

A model for a nonlinearity-dominated wind-wave system has been proposed. Results of laboratory experimental investigations of nonlinear wave trains and wind waves have been examined and found to provide evidence supporting the proposed physical model. The following is a review of conclusions which we believe can be drawn at this time regarding the hydrodynamic characteristics of nonlinear wind waves and a brief discussion of possible further applications of the wind-wave model.

For developing wind waves with steady wind and waves in one direction, we find that:

(i) The spectral components of a nonlinearity-dominated wind-wave system are effectively non-dispersive Fourier components bound to the dominant wave and are not a random collection of free waves each obeying the usual dispersion relation.

(ii) At a fixed fetch, the observed properties of nonlinear wind-driven waves strongly support an interpretation of the waves as a coherent bound-wave system characterized by a single dominant frequency (which may vary slowly in time within a narrow frequency range), propagating energy at a single speed, and characterized by nonlinear self-interactions of the type found in strongly amplitude-modulated nonlinear wave trains.

(iii) The bound-wave nature of such a nonlinear wind-wave system implies that the dominant wave alone receives almost all of the energy input from the wind. Changes in the bound frequency components merely reflect changes in the dominant wave form, and should not be identified as changes induced by direct wind energy input to those components.

(iv) The evolution of a nonlinear wind-wave system to progressively lower dominant wavenumber and frequency, as the system gains energy from the wind, is a consequence of hydrodynamic self-interaction and a limiting wave steepness condition for deep-water gravity waves. The hydrodynamic interaction which produces the frequency/wavenumber shift has been observed in 'over-modulated' nonlinear wave trains (i.e. without wind) and measured in detail. For wave trains the interaction occurs among components having frequency separation $\Delta f = \bar{k}\bar{a}f$, and the magnitude of the decrease in carrier frequency during one frequency shift interaction is also Δf. Although the wave-train measurements clearly demonstrate that this self-interaction produces a true change in carrier frequency, the interaction has not been predicted by existing theories for wave-train interactions. We believe that the interaction as it has been identified in the wave-train case suggests an essential interaction process by which the dominant wave in a nonlinear wind-wave system evolves to lower frequency and wavenumber as it gains energy from the wind.

(v) The process of wind-wave evolution, as modelled in terms of a dominant nonlinear wave train which continually gains energy, approaches limiting steepness, shifts to lower frequency, and gains still more energy, etc. is consistent with the proposition that the dominant wave steepness $\bar{k}\bar{a}$ is approximately constant as a function of fetch or duration for fixed wind speed and $u_w/C \gg 1$ in an actively growing wind-wave system. This constant steepness condition is also consistent with measurements of dominant wave steepness in such systems and can be used to predict the evolution of the dominant wave frequency with fetch or duration as a nonlinear wind-wave system gains wave energy or amplitude from the wind. The condition of constant dominant wave steepness is equivalent to an $\bar{a}^2/f \propto f^{-5}$ relationship between the magnitudes of successive spectral peaks in a series of spectra for evolving nonlinear wind waves.

(vi) The bound-wave components of the dominant wave in a nonlinear wind-wave system contain essentially all of the wave energy, but the wind-wave system also includes high-frequency free waves (which obey the dispersion relation) that are locally generated by wind or wave breaking and that exist along the surface of the dominant wave. Although they contain a negligible fraction of the total energy in the wind-wave system, the short free waves are important in that they are likely to be strongly affected by, and strongly coupled to, the characteristics of the dominant wave on which they exist.

(vii) Wind waves generated in laboratory facilities are coherent nonlinear bound-wave systems of the type described by our model. This model is also expected to apply to oceanic wind waves when the waves can be characterized hydrodynamically as having a one-directional coherent dominant wave with an average steepness greater than about 0·1. A preliminary examination of oceanic wave data indicates that the application of the model may not necessarily be limited simply to wind-wave systems with short fetch or duration. On the other hand, under many circumstances the ocean state may be too linear (too flat, $\bar{k}\bar{a} < 0{\cdot}1$), too multi-directional, or too multi-sourced for this model to apply. Continued examination of oceanic wave data is required in order to assess in full the applicability of this nonlinear wind-wave model to oceanic wind waves.

(viii) When wind waves are nonlinear in the sense of this model, the statistical properties of the wave modulations, as measured by the wave envelopes, should be related to the statistical properties of the dominant waves in the same way that modulational and carrier-wave properties are related in the case of nonlinear wave trains. These results suggest that greater attention should be given to modulational characteristics in future analyses of wind-wave measurements.

(ix) Although there is evidence that in the sense of average properties (such as relative levels of harmonic components in spectra), nonlinear wind waves may have Stokes-wave-like properties, detailed measurements of nonlinear wave trains and wind waves show that because these wave systems are necessarily highly modulated the waves are far too unsteady to be well modelled, either individually or in groups, using detailed properties of Stokes waves. Our investigations of nonlinear wave trains and wind waves indicate to us that the most appropriate model for such systems is one which describes the system in terms of the dynamics of the dominant-wave envelope using the nonlinear Schrödinger equation as a basis. In order to identify properties of individual carrier waves correctly, an envelope solution and an analysis of unsteady deep-water gravity waves are required.

(x) As a first step toward exploiting this new interpretation theoretically, a first-order theory for the evolution of nonlinear wind waves has been proposed by Yuen & Lake (1976). Since the rate of input of wind energy into the waves is slow compared with the rate of modulation caused by self-nonlinearity, the wind-wave field is characterized as an adiabatic system consisting of a nonlinear wave train with a slowly-varying carrier frequency. The adiabatic nature of the wave field strongly favours the use of a wind-wave energy input model which 'tracks' the frequency change of the dominant wave, as opposed to the fixed mode approach of Miles (1957, 1959*a*, *b*, 1960, 1964) and Phillips (1957, 1958*b*). The first-order theoretical model of Yuen & Lake (1976) employs the nonlinear Schrödinger equation for the wave envelope, supplemented by an equation describing the change of dominant-wave frequency (or wavenumber) with fetch (or duration). The latter equation is a consequence of a form drag model proposed by Deardorff (1967) for the input of energy into the dominant wave, which in turn implies that the wave drag coefficient can be assumed to be a function of averaged dominant-wave slope only and that the averaged dominant-wave slope is constant during a significant portion of the wave growth.

(xi) Although the results presented here are limited to one-dimensional wave-train and wind-wave systems, there is evidence that the basic physical properties of these nonlinear wave systems are relevant to two-dimensional systems as well. The evolution of nonlinear wave trains in two space dimensions has been studied by Zakharov (1968), who derived the appropriate two-space-dimensional nonlinear Schrödinger equation for the complex wave envelope. Zakharov (1968), Zakharov & Rubenchik (1974), and Saffman & Yuen (1978) used the equation to study stability properties of nonlinear wave trains in two space dimensions. The results of those investigations, together with numerical solutions of the equation obtained by Yuen & Ferguson (1979), indicate that even though the structure of the solutions is far more complicated than in the one-space-dimensional case, the most important features of a nonlinear wave train, namely, modulational instability, Fermi–Pasta–Ulam recurrence and coherence, are still present in two space dimensions. The application of these results to the formulation of a first-order nonlinear wind-wave model in two dimensions is considered in Yuen & Lake (1979).

(xii) One important feature of our proposed model for nonlinear wind waves is that the effects of nonlinearity on the dynamics of the wave system are predominant over the effects of randomness. This property has allowed us to propose a deterministic system for the description of the physical mechanisms associated with the dominant waves. To obtain information concerning the statistical properties of nonlinear wind-wave systems, we are presently examining the properties of random phase solutions of the nonlinear Schrödinger equation. A first step along this line of investigation has been taken by Alber & Saffman (1977) and Crawford, Saffman & Yuen (1977), who have examined the stability of random phase nonlinear wave trains with particular statistical distributions.

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